# Theory of the AC Hall effect in polycrystalline semiconductors



1994 J. Phys.: Condens. Matter 6 2747

(http://iopscience.iop.org/0953-8984/6/14/012)

J. Phys.: Condens. Matter 6 (1994) 2747–2750. Printed in the UK

# Theory of the AC Hall effect in polycrystalline semiconductors

I I Fishchuk

Institute for Nuclear Research, Ukrainian Academy of Sciences, Kiev, Ukraine

Received 2 November 1993

**Abstract.** The theory of the AC Hall effect for systems which consist of randomly oriented anisotropic crystallites is developed. It is shown that the real parts of the dynamic conductivity, the Hall conductivity, the Hall mobility and the Hall coefficient have low- and high-frequency plateaux. The high-frequency plateau begins in the vicinity of the frequency $\omega_0 \simeq \tau_M^{-1}$, where $\tau_M$ is the Maxwell relaxation time. The imaginary parts of these quantities have sharp peaks at the frequency $\omega_0$.

## 1. Introduction

The AC kinetic phenomena in inhomogeneous solids have been investigated theoretically in many papers. In the majority of these studies the effective-medium theory (EMT) was used. The validity of the EMT results was corroborated in many cases by comparison with numerical simulation data. The AC Hall effect and magnetoresistance in semiconductors with random low-conductivity inclusions were investigated by Fishchuk (1983, 1986, 1989, 1992). The Hall effect results (Fishchuk 1983, 1986) were used by Jaouen *et al* (1986, 1989) to interpret the experimental data on the frequency dependences of the Hall mobility in silicon implanted with arsenic ions.

In the present paper we have studied a polycrystalline material which consists of randomly oriented anisotropic semiconductor crystallites. A theory of the DC Hall effect for such systems was developed by Xia and Stroud (1988) using the EMT. We have extended this theory to the AC Hall effect and have investigated the frequency dependences of the effective conductivity, the Hall conductivity, the Hall mobility and the Hall coefficient.

## 2. Theory

Let us consider a polycrystalline sample which consists of randomly oriented anisotropic semiconductor crystallites. The average crystallite size is much smaller than the sample size. Therefore, in the absence of a magnetic field, the sample is isotropic. We study, as in the above-cited papers by Fishchuk, the frequency range, where $\omega < \tau^{-1}$. Here $\tau$ is the electron mean free time in each crystallite. We suppose, as did Xia and Stroud (1988), that $\tau$ has neither energy dispersion nor anisotropy. When the displacement current is taken into account, the complex conductivity tensor $\hat{\sigma}^c$ in the crystallite and the effective conductivity tensor $\hat{\sigma}^c_e$ of sample may be written as $\hat{\sigma}^c = \hat{\sigma} + i\omega\mathcal{E}_0\hat{I}/4\pi$ and $\hat{\sigma}^c_e = \hat{\sigma}_e + i\omega\mathcal{E}_0\hat{I}/4\pi$. Here $\mathcal{E}_0$ is the scalar dielectric constant in each crystallite. The value $\hat{\sigma}_e$ has an imaginary part due to the inhomogeneity of the sample. We use the EMT as did Xia and Stroud (1988). Let a weak external magnetic field $H$ along the sample axes have the components $\{0, 0, H\}$.

We suppose that the principal axes of the crystallites are randomly oriented with equal probability in each direction. Along the principal axes the components of the magnetic field are $\{H_x, H_y, H_z\}$, and $\hat{\sigma} = \hat{\sigma}_s + \hat{\sigma}_a$, where

$$\hat{\sigma}_s = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \qquad \hat{\sigma}_a = \begin{bmatrix} 0 & \sigma_{xy} & -\sigma_{zx} \\ -\sigma_{xy} & 0 & 0 \\ \sigma_{zx} & -\sigma_{yz} & 0 \end{bmatrix}. \tag{1}$$

The values $\sigma_{xy}$, $\sigma_{yz}$ and $\sigma_{zx}$ may be written as $\sigma_{ij} = \gamma_{kk} H_k$, where $\gamma_{kk} = \sigma_{ii}\sigma_{jj}/nec$, and $n$ is the charge carrier concentration. The effective conductivity tensor $\hat{\sigma}_e$ can be written $\hat{\sigma}_e = \hat{\sigma}_e^s + \hat{\sigma}_e^a$, where

$$\hat{\sigma}_e^s = \sigma_e^s \hat{I} \qquad \hat{\sigma}_e^a = \sigma_e^a \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{2}$$

We have calculated $\hat{\sigma}_e$ using the EMT. The appropriate EMT equation has the form (Stroud 1975, Fishchuk 1992)

$$\left\langle (\hat{I} - \Delta\hat{\sigma}\hat{\Gamma})^{-1} \cdot \Delta\hat{\sigma} \right\rangle = 0 \tag{3}$$

where $\Delta\hat{\sigma} = \Delta\hat{\sigma}_s + \Delta\hat{\sigma}_a$, $\Delta\hat{\sigma}_s = \hat{\sigma}_s' - \hat{\sigma}_e^s$, $\Delta\hat{\sigma}_a = \hat{\sigma}_a' - \hat{\sigma}_e^a$, $\hat{\Gamma} = -\hat{I}/3(\sigma_e^s + i\omega\mathcal{E}_0/4\pi)$ and $\hat{\sigma}_s'$ and $\hat{\sigma}_a'$ are conductivities in the random oriented crystallite in the sample axes. We assume that the crystallites can be treated as approximately spherical (Xia and Stroud 1988). The angular brackets in (3) denote the orientation averaging. Further we expand (3) in powers of $H$, taking into account only the zeroth and first powers. Equating with zero each averaged term of series we obtain the system of equations for calculating the values $\sigma_e^s$ and $\sigma_e^a$ in the form

$$\left\langle A_{xx}^{-1}(\sigma_{xx}' - \sigma_e^s) \right\rangle = 0 \qquad \left\langle A_{xx}^{-1}(\sigma_{xy}' - \sigma_e^a) A_{yy}^{-1} \right\rangle = 0 \tag{4}$$

where $A_{ii} = \sigma_{ii} + 2\sigma_e^s + i\omega 3\mathcal{E}_0/4\pi$. Averaging the equations (4) over the orientations, we obtain

$$\sum_{i=1}^{3} A_{ii}(\sigma_{ii} - \sigma_e^s) = 0 \qquad \sigma_e^a = \left( \sum_{i=1}^{3} A_{ii}\gamma_{ii} \Big/ \sum_{i=1}^{3} A_{ii} \right) H. \tag{5}$$

Here 1, 2, 3 denote $X$, $Y$, $Z$. Further calculations of $\sigma_e^s$ and $\sigma_e^a$ are possible only after choice of the ratio between diagonal components of $\hat{\sigma}_s$.

## 3. Frequency dependences of the Hall effect quantities

We have applied equations (5) to the case when $\sigma_{xx} = \sigma_{yy} \gg \sigma_{zz}$. Under this condition we have $\gamma_{xx} = \gamma_{yy} \ll \gamma_{zz}$. This relaxation may be realized, for example, in the oxides $La_{2-x}M_xCuO_4$ (M = Sr or Ba) (Xia and Stroud 1988) because of an anisotropic mass tensor of carriers ($m_x = m_y \ll m_z$). Then keeping the lowest order of the ratio $\sigma_{zz}/\sigma_{xx}$ in equations (5), we find that

$$(\sigma_e^s)^2 - \frac{1}{2}\sigma_e^s \left( \sigma_{xx} - i\omega \frac{3\mathcal{E}_0}{4\pi} \right) - i\omega \frac{\mathcal{E}_0}{4\pi}\sigma_{xx} = 0 \tag{6}$$

$$\sigma_e^a = \frac{2\sigma_e^s + i\omega(3\mathcal{E}_0/4\pi)}{2\sigma_{xx} + 6\sigma_e^s + i\omega(9\mathcal{E}_0/4\pi)} \sigma_{xy}. \tag{7}$$

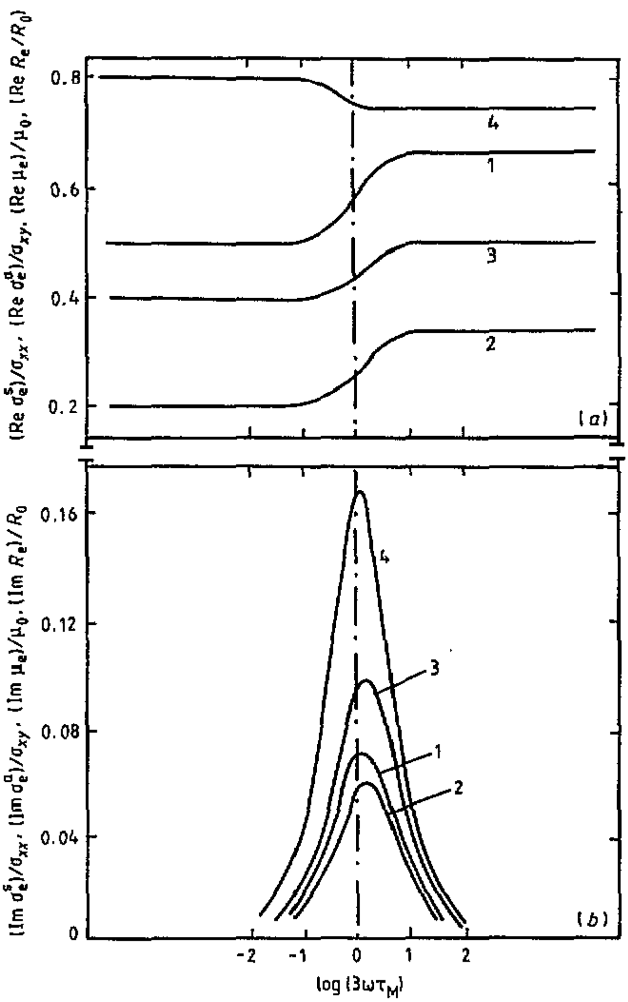


**Figure 1.** Frequency dependences of (*a*) Re $\sigma_e^s$ (curve 1), Re $\sigma_e^a$ (curve 2), Re $\mu_e$ (curve 3) and Re $R_e$ (curve 4) and (*b*) Im $\sigma_e^s$ (curve 1), Im $\sigma_e^a$ (curve 2), Im $\mu_e$ (curve 3) and Im $R_e$ (curve 4) in polycrystalline semiconductors.

For the physical solution $\sigma_e^s = \mathrm{Re}\,\sigma_e^s + \mathrm{i}\,\mathrm{Im}\,\sigma_e^s$ of equation (6), we have

$$\mathrm{Re}\,\sigma_e^s = \tfrac{1}{4}\sigma_{xx} + \tfrac{1}{4}\sigma_{xx}(a^2+b^2)^{1/4}\begin{cases}\cos\varphi \\ -\sin\varphi\end{cases} \quad \text{for} \begin{cases}3\omega\tau_M < 1 \\ 3\omega\tau_M > 1\end{cases} \tag{8}$$

$$\mathrm{Im}\,\sigma_e^s = -\tfrac{1}{4}\sigma_{xx}3\omega\tau_M + \tfrac{1}{4}\sigma_{xx}(a^2+b^2)^{1/4}\begin{cases}\sin\varphi \\ \cos\varphi\end{cases} \quad \text{for} \begin{cases}3\omega\tau_M < 1 \\ 3\omega\tau_M > 1\end{cases} \tag{9}$$

where $\varphi = \frac{1}{2}[\tan^{-1}(b/a)]$, $a = 1 - (3\omega\tau_M)^2$, $b = 10\omega\tau_M$ and $\tau_M = \mathcal{E}_0/4\pi\sigma_{xx}$ is the Maxwell relaxation time ($\tau_M \gg \tau$). Inserting the solution $\sigma_e^s$ into equation (7) we find the value $\sigma_e^a = \mathrm{Re}\,\sigma_e^a + \mathrm{i}\,\mathrm{Im}\,\sigma_e^a$. In the limit $\omega \to 0$ we have the results of Xia and Stroud: $\sigma_e^s = \frac{1}{2}\sigma_{xx}$, $\sigma_e^a = \frac{1}{5}\sigma_{xy}$. In this case the Hall mobility $\mu_e$ and the Hall coefficient $R_e$ take the forms $\mu_e = \frac{2}{5}\mu_0$, $R_e = \frac{4}{5}R_0$, $\mu_0 = e\tau/m_x$ and $R_0 = 1/nec$. In the opposite case of high frequencies, when $\tau_M^{-1} \ll \omega < \tau^{-1}$ we obtain the expressions $\sigma_e^s = \frac{2}{3}\sigma_{xx}$, $\sigma_e^a = \frac{1}{3}\sigma_{xy}$, $\mu_e = \frac{1}{2}\mu_0$, $R_e = \frac{3}{4}R_0$.

In figure 1 the frequency dependences of the functions $(\mathrm{Re}\,\sigma_e^s)/\sigma_{xx}$, $(\mathrm{Re}\,\sigma_e^a)/\sigma_{xy}$, $(\mathrm{Re}\,\mu_e)/\mu_0$, $(\mathrm{Re}\,R_e)/R_0$, $(\mathrm{Im}\,\sigma_e^s)/\sigma_{xx}$, $(\mathrm{Im}\,\sigma_e^a)/\sigma_{xy}$, $(\mathrm{Im}\,\mu_e)/\mu_0$ and $(\mathrm{Im}\,R_e)/R_0$ obtained from equations (7)–(9) are shown. Here $\mu_e = -(C/H)\sigma_e^a/\mathrm{Re}\,\sigma_e^s$ and $R_e = (1/H)\sigma_e^a/(\mathrm{Re}\,\sigma_e^s)^2$.

## 4. Discussion of the results

The theory of the AC Hall effect in polycrystalline materials which consist of randomly oriented anisotropic semiconductor crystallites has been developed using the EMT. We have investigated the frequency range $\omega < \tau^{-1}$. Equations for calculation of the AC complex effective conductivity $\sigma_e^s(\omega)$, the Hall conductivity $\sigma_e^a(\omega)$, the Hall mobility $\mu_e(\omega)$ and the Hall coefficient $R_e(\omega)$ are obtained. Calculations have been performed for highly anisotropic crystallites.

The results show that in both the low-frequency ($\omega \ll \tau_M^{-1}$) and the high-frequency ($\tau_M^{-1} \ll \omega < \tau^{-1}$) ranges we obtain plateaux in the values of Re $\sigma_e^s(\omega)$, Re $\sigma_e^a(\omega)$, Re $\mu_e(\omega)$ and Re $R_e(\omega)$. In the intermediate range ($\omega \simeq \tau_M^{-1}$) the values of Re $\sigma_e^s(\omega)$, Re $\sigma_e^a(\omega)$ and Re $\mu_e(\omega)$ increase and the value of Re $R_e(\omega)$ decreases with increase in $\omega$. In this range the values of Im $\sigma_e^s(\omega)$, Im $\sigma_e^a(\omega)$, Im $\mu_e(\omega)$ and Im $R_e(\omega)$ have sharp peaks.

Similar results were obtained for the AC Hall conductivity and mobility in inhomogeneous semiconductors (Fishchuk 1983, 1986) and in disordered solids with hopping conductivity (Movaghar *et al* 1983). These results (Fishchuk 1983, 1986) were used to interpret experimental data (Jaouen *et al* 1986, 1989) for disordered silicon.

## References

Fishchuk I I 1983 *Fiz. Tekh. Poluprov.* **17** 1189–94
—— 1986 *Phys. Status Solidi* a **93** 675–84
—— 1989 *Fiz. Tverd. Tela* **31** 135–9
—— 1992 *J. Phys.: Condens. Matter* **4** 8045–52
Jaouen H, Ghibaudo G and Christofidés C 1986 *J. Appl. Phys.* **60** 1699–704
—— 1989 *J. Appl. Phys.* **65** 4832–9
Movaghar B, Pohlmann B and Würtz D 1983 *J. Phys. C: Solid State Phys.* **16** 3755–62
Stroud D 1975 *Phys. Rev.* B **12** 3368–73
Xia T K and Stroud D 1988 *Phys. Rev.* B **37** 118–22